#### Binomial Expansion - Year 1 Core

1 (a)	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5 x^5$	M1	1.1a
	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5 x^5$	A1	1.1b
	Sets $448a^5 = 3402 \implies a^5 = \frac{243}{32}$	M1	1.1b
	$\Rightarrow a = \frac{3}{2}$	A1	1.1b
		(4)	
<b>(b)</b>	Attempts either term. So allow for $2^8$ or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	= 256 + 5670 = 5926	A1	1.1b
		(3)	
	(7 ma		

#### Notes

**(a)** 

**M1:** An attempt at selecting the correct term of the binomial expansion. If all terms are given then the correct term must be used. Allow with a missing bracket  ${}^{8}C_{5}2^{3}ax^{5}$  and left without the binomial coefficient expanded

A1:  $448a^5x^5$  Allow unsimplified but  ${}^{8}C_5$  must be "numerical"

**M1:** Sets their  $448a^5 = 3402$  and proceeds to  $\Rightarrow a^k = \dots$  where  $k \in \mathbb{N}$   $k \neq 1$ 

**A1:** Correct work leading to  $a = \frac{3}{2}$ 

# **(b)**

**M1:** Finds either term required. So allow for  $2^8$  or  ${}^8C_4 2^4 a^4$  (even allowing with *a*)

**dM1:** Attempts the sum of both terms  $2^8 + {}^8C_4 2^4 a^4$ 

A1: cso 5926

Question	Scheme	Marks	AOs	
2(a)	$3^8$ or 6561 as the constant term	B1	1.1b	
	$\left(3 - \frac{2x}{9}\right)^8 = \dots + {}^8C_1 \left(3\right)^7 \left(-\frac{2x}{9}\right) + {}^8C_2 \left(3\right)^6 \left(-\frac{2x}{9}\right)^2 + {}^8C_3 \left(3\right)^5 \left(-\frac{2x}{9}\right)^3 + \dots$ $= \dots + 8 \times \left(3\right)^7 \left(-\frac{2x}{9}\right) + 28 \times \left(3\right)^6 \left(-\frac{2x}{9}\right)^2 + 56 \left(3\right)^5 \left(-\frac{2x}{9}\right)^3$	M1 A1	1.1b 1.1b	
	$= 6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \dots$	Al	1.1b	
		(4)		
(b)	Coefficient of $x^2$ is $\frac{1}{2} \times "1008" - \frac{1}{2} \times " - \frac{448}{3}"$	M1	3.1a	
	$=\frac{1736}{3}$ (or 578 $\frac{2}{3}$ )	A1	1.1b	
		(2)		
		(6 marks		
	Notes			

- B1: Sight of  $3^8$  or 6561 as the constant term.
- M1: An attempt at the binomial expansion. This can be awarded for the correct structure of the 2<sup>nd</sup>, 3<sup>rd</sup> or 4<sup>th</sup> term. The correct binomial coefficient must be associated with the correct power of 3 and the correct power of  $(\pm)\frac{2x}{9}$ . Condone invisible brackets

eg 
$${}^{8}C_{2}(3)^{6} - \frac{2x^{2}}{9}$$
 for this mark.

A1: For a correct simplified or unsimplified **second** or **fourth term** (with binomial coefficients evaluated).

$$+8 \times (3)^7 \left(-\frac{2x}{9}\right)$$
 or  $+56(3)^5 \left(-\frac{2x}{9}\right)^3$ 

A1:  $6561-3888x+1008x^2 - \frac{448}{3}x^3$  Ignore any extra terms and allow the terms to be listed. Allow the exact equivalent to  $-\frac{448}{3}$  eg -149.3 but not -149.3. Condone  $x^1$  and eg +-3888x. Do not isw if they multiply all the terms by eg 3 Alt(a)

- B1: Sight of  $3^8(1+....)$  or 6561 as the constant term
- M1: An attempt at the binomial expansion  $\left(1-\frac{2}{27}x\right)^8$ . This can be awarded for the correct structure of the 2<sup>nd</sup>, 3<sup>rd</sup> or 4<sup>th</sup> term. The correct binomial coefficient must be associated with the correct power of  $(\pm)\frac{2x}{27}$ . Condone invisible brackets for this mark.

$$8 \times -\frac{2}{27}x, \quad \frac{8 \times 7}{2} \times \left(-\frac{2}{27}x\right)^2, \quad \frac{8 \times 7 \times 6}{6} \times \left(-\frac{2}{27}x\right)^3 \text{ which may be implied by any of} \\ -\frac{16}{27}x, \quad +\frac{112}{729}x^2, \quad -\frac{448}{19683}x^3$$

- A1: For a correct simplified or unsimplified **second** or **fourth** term including being multiplied by 3<sup>8</sup>
- A1:  $6561-3888x+1008x^2 \frac{448}{3}x^3$  Ignore any extra terms and allow the terms to be listed. Allow the exact equivalent to  $-\frac{448}{3}$  eg -149.3 but not -149.3. Condone  $x^1$  and eg +-3888x

# (b)

M1: Adopts a correct strategy for the required coefficient. This requires an attempt to calculate  $\pm \frac{1}{2}$  their coefficient of  $x^2$  from part (a)  $\pm \frac{1}{2}$  their coefficient of  $x^3$  from part (a).

There must be an attempt to bring these terms together to a single value. ie they cannot just circle the relevant terms in the expansion for this mark. The strategy may be implied by their answer.

Condone any appearance of  $x^2$  or  $x^3$  appearing in their intermediate working.

A1: 
$$\frac{1736}{3}$$
 or  $578\frac{2}{3}$  Do not accept  $578.6$  or  $\frac{1736}{3}x^2$ 

Questi	on Scheme	Marks	AOs	
3	Attempts the term in $x^3$ or the term in $x^5$ of $\left(3 - \frac{1}{2}x\right)^6$	M1	3.1a	
	Look for ${}^{6}C_{3}3^{3}\left(-\frac{1}{2}x\right)^{3}$ or ${}^{6}C_{5}3^{1}\left(-\frac{1}{2}x\right)^{5}$		5.1a	
	Correct term in $x^3$ or correct term in $x^5$ of $\left(3 - \frac{1}{2}x\right)^6$ $-\frac{135}{2}x^3$ or $-\frac{9}{16}x^5$	A1	1.1b	
	Attempts one of the required terms in $x^5$ of $(5+8x^2)(3-\frac{1}{2}x)^6$ Either $5 \times {}^6\text{C}_5 3^1(-\frac{1}{2}x)^5$ or $8x^2 \times {}^6\text{C}_3 3^3(-\frac{1}{2}x)^3$	M1	1.1b	
	Attempts the sum of $5 \times {}^{6}C_{5}3^{1}\left(-\frac{1}{2}x\right)^{5}$ and $8x^{2} \times {}^{6}C_{3}3^{3}\left(-\frac{1}{2}x\right)^{3}$	dM1	2.1	
	Coefficient of $x^5 = -\frac{45}{16} - 540 = -\frac{8685}{16}$	A1	1.1b	
		(5)		
		(5 n	narks)	
Notes:				
	For the key step in attempting to find one of the required terms in the expan	sion of		
	$\left(3-\frac{1}{2}x\right)^{6}$ to enable the problem to be solved.			
]	Look for ${}^{6}C_{3}3^{3}\left(-\frac{1}{2}x\right)^{3}$ or ${}^{6}C_{5}3^{1}\left(-\frac{1}{2}x\right)^{5}$ but condone missing brackets ar	nd slips in s	igns.	
	May be part of a complete expansion but only one of the required terms nee correct form.	ds to be of	the	
	For $-\frac{135}{2} \{x^3\}$ or $-\frac{9}{16} \{x^5\}$ which may be unsimplified but the ${}^6C_3$ or ${}^6C_5$ must be			
]	processed. May be implied by $-540\left\{x^{5}\right\}$ or $-\frac{45}{16}\left\{x^{5}\right\}$	. 6		
M1:	Attempts one of the required terms in $x^5$ of the expansion of $(5+8x^2)(3-\frac{1}{2})$	$\left(\frac{1}{2}x\right)^{0}$		
]	Look for $5 \times {}^{6}C_{5}3^{1}\left(-\frac{1}{2}x\right)^{5}$ or $8x^{2} \times {}^{6}C_{3}3^{3}\left(-\frac{1}{2}x\right)^{3}$ which would also imply	y the previo	ous M.	
	The $x^5$ may be missing as just the coefficient is required.			
]	May be implied by $-540 \{x^5\}$ or $-\frac{45}{16} \{x^5\}$			
	Condone missing brackets and signs. You might see candidates make a slip in, e.g., their binomial coefficients, but (essentially) correct method to solve the problem	ut have an		

(essentially) correct method to solve the problem.

Note that this M mark is not dependent on the first, so you may be able to award it even if they have made a slip in finding their  $x^3$  or  $x^5$  term in the expansion.

**dM1:** Attempts the sum of  $5 \times {}^{6}C_{5}3^{1}\left(-\frac{1}{2}x\right)^{5}$  and  $8x^{2} \times {}^{6}C_{3}3^{3}\left(-\frac{1}{2}x\right)^{3}$ 

Dependent on the previous M but may be scored at the same time.

The  $x^5$  may be missing as just the coefficients are required. Condone missing brackets and signs.

A1: 
$$-\frac{8685}{16}$$
 or exact equivalent, -542.8125 and apply isw  
Condone  $-\frac{8685}{16}x^5$  for A1  
Note that rounded decimals, e.g., -542.81 will not score the last mark.

Note that full marks can be scored for concise solutions such as:

$$5 \times {}^{6}C_{5} \times 3 \times \left(-\frac{1}{2}\right)^{5} + 8 \times {}^{6}C_{3} \times 3^{3} \times \left(-\frac{1}{2}\right)^{3} = -\frac{8685}{16}$$

### Alternative

Attempts via the taking out of the common factor can be scored in the same way.

$$\left(3 - \frac{1}{2}x\right)^{6} = 3^{6} \left\{1 + 6 \times \left(-\frac{1}{6}x\right)^{1} + \frac{6 \times 5}{2} \left(-\frac{1}{6}x\right)^{2} + \frac{6 \times 5 \times 4}{3!} \left(-\frac{1}{6}x\right)^{3} + \frac{6 \times 5 \times 4 \times 3}{4!} \left(-\frac{1}{6}x\right)^{4} + \frac{6 \times 5 \times 4 \times 3 \times 2}{5!} \left(-\frac{1}{6}x\right)^{5} + \left(-\frac{1}{6}x\right)^{6}\right\}$$
  
For M1 A1 look for  $3^{6} \times \frac{6 \times 5 \times 4}{3!} \left(-\frac{1}{6}x\right)^{3}$  or  $3^{6} \times \frac{6 \times 5 \times 4 \times 3 \times 2}{5!} \left(-\frac{1}{6}x\right)^{5}$   
Score the remaining marks as per the main scheme.